Discussing proof in STEM fields

Math and Science teachers' use of inductive evidence

Nick Wasserman
Dara Williams-Rossi
Southern Methodist University

INTRO

- STEM (Science, Technology, Engineering, Mathematics) has become increasingly emphasized in education
- Yet the interpretation and implementation of what STEM education means in practice, varies widely.

STEM

- National Science Teachers Association (NSTA) reports:

"Everybody…knows what [STEM] means within their field, and everybody else is defining it to fit their own needs. Whether it is researchers, science and mathematics teachers, the aerospace industry, or the construction industry, they all have one thing in common: It is about moving forward, solving problems, learning, and pushing innovation to the next level."

STEM

California Department of Education

- "A nationally agreed upon definition for STEM education is currently lacking"…"**Could be a stand alone course, a sequence of courses, activities involving any of the four areas**, a STEM-related course, or an interconnected or integrated program of study."
- Implementation of STEM, according to this definition, could mean anything from enhancing individual content areas or deeper cross-disciplinary integration

STEM

- California STEM Learning Network (CSLNet) believes that STEM education is more than just science, technology, engineering or mathematics; it is an interdisciplinary and applied approach that is coupled with real-world, problem-based learning. This bridging among the four discrete disciplines is now known as STEM. STEM education **removes the traditional barriers erected between the four disciplines by integrating them into one cohesive teaching and learning paradigm.**

STEM

- Dayton Regional Stem Center, STEM Ed Quality Framework, includes:
  - **Degree of STEM Integration**: Quality STEM learning experiences are carefully designed to help students integrate knowledge and skills from Science, Technology, Engineering, and Mathematics.
  - **Integrity of the Academic Content**: Quality STEM learning experiences are content-accurate, anchored to the relevant content standards, and focused on the big ideas and foundational skills critical to future learning in the targeted discipline(s).

STEM

- The result of a 2008 study on promising practices on undergraduate STEM education lead to the development of Discipline-Based Education Researcher (DBER).
- Based on this work particularly across 4 science fields: physics, biology, geoscience and chemistry, **the premise of DBER is that teaching and learning of these subjects requires deep discipline specific knowledge.**
- This poses some tension between STEM **integration** and **content integrity**.

Reasoning in Mathematics

- Reasoning and sense-making in mathematics (NCTM)
  - Mathematics education should be focused on students reasoning and sense-making
  - There are many valid forms of reasoning about mathematics
- Deductive reasoning and formal proof, however, are standard for adding new knowledge to the field; axioms, definitions, logical arguments, proof

Proof in Mathematics

- Many have studied and debated what role proof should play in mathematics education (e.g., Chazan, 1993; Hanna, 1995; Knuth, 2002; Stylianides, 2007; etc.)
- As a part of some of this work, there is a general taxonomy for proof schemes:
  - External Conviction
  - Empirical (example-based evidence)
  - Deductive

Proof in Mathematics

- Balacheff (1988) further expanded on this taxonomy:
  - Naïve empiricism (small number of particular examples)
  - Crucial experiment (after particular examples, examines non-particular case)
  - Generic example (example is representative of a class)
  - Thought experiment (logical deductions)

Proof in Mathematics

- Harel & Sowder (1998)
  - External conviction
  - Empirical proof scheme
    - Inductive
    - Perceptual
  - Analytical proof scheme
    - Transformational
      - Restrictive – generic
      - Internalized/Interiorized (non-restrictive)
    - Axiomatic

Reasoning in Science

- Observation
- Repeated trials
- Generalizability

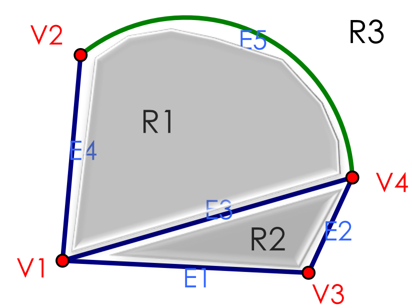
Sample Problem

What type of reasoning might you engage in to determine if the following claim is true?

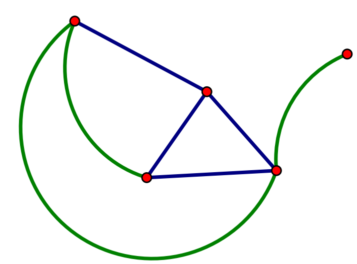
Bob draws some "diagrams" where no edges (curved or straight) intersect each other. (Also, there are not two "separate" diagrams.) He claims that if you count the regions, R, created by the V vertices and E edges (including the "outside" region), that

R = E + 2 – V, always.

Example Bob diagram



V=4, E=5, and it makes R=3

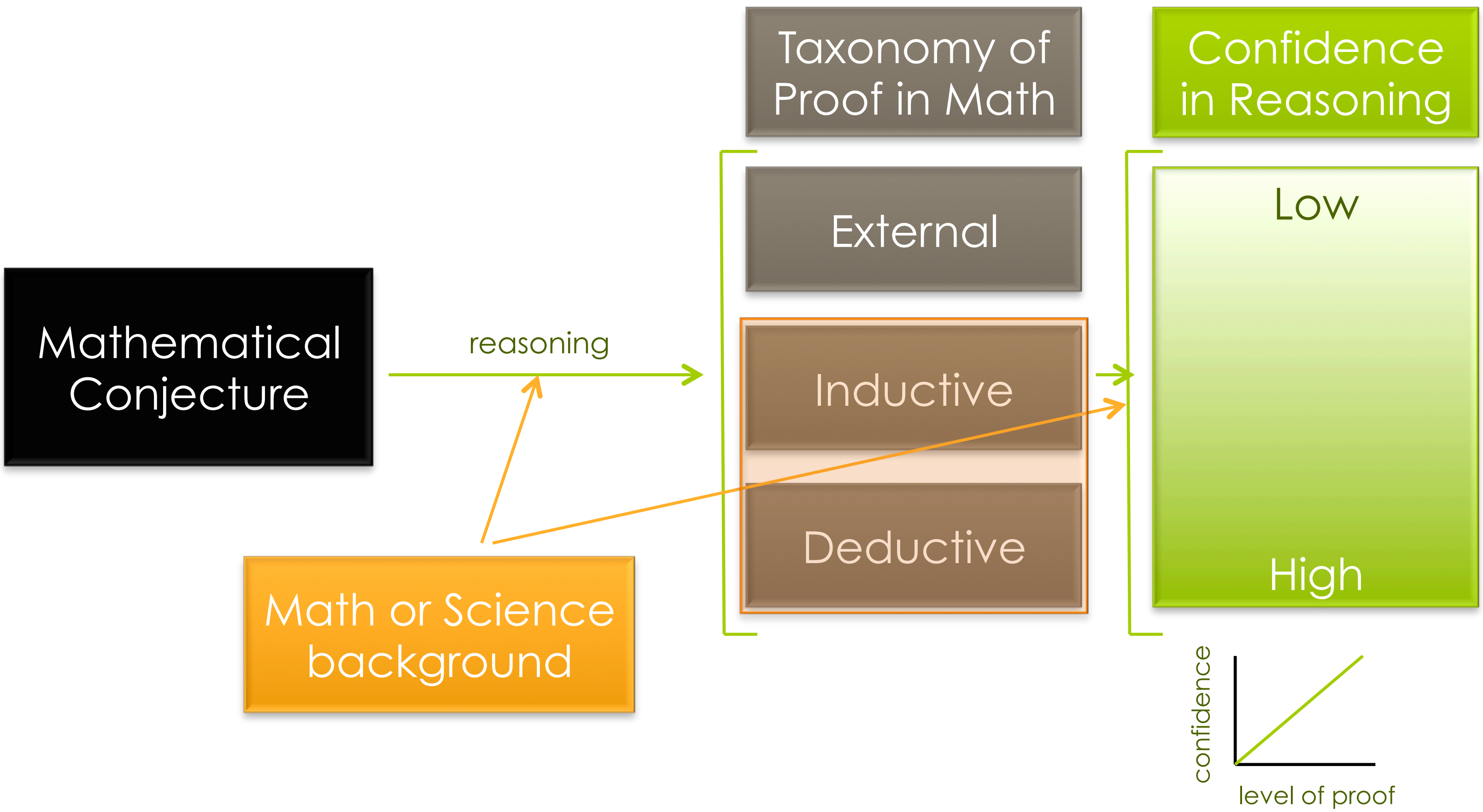


V=5, E=7, and it makes R=4

Research Question

- Given the current trend toward integration of STEM disciplines, and the distinct forms of reasoning in mathematics and science, we asked the following research questions:
  - Do math and science teachers reason differently to validate mathematical ideas – in particular, does reliance on empirical/inductive evidence impact their level of confidence in their validation and reasoning?
  - Do math and science teachers identify any distinction between the primary modes of reasoning in each discipline?

Framework


Mathematical Conjecture
reasoning
Math or Science background
Taxonomy of Proof in Math
External
Inductive
Deductive
Confidence in Reasoning
Low
High
confidence
level of proof

Methodology

- Participants
  - STEM teachers
  - Majority Graduate students

Math n=24		Science n=23	
17 middle school	4 male	7 middle school	10 male
7 high school	20 female	16 high school	13 female
14 math/math education degree		18 science degree	

Creation of Tasks

- In order to disentangle whether mathematics and science teachers engage differently in reasoning, and have different degrees of confidence in the sufficiency of inductive reasoning, 3 tasks were created so that inductive reasoning would likely be the logical first step.

Tasks

For each of the following claims, justify whether of not you believe Bob's statement to be true or not by citing evidence and discussing your reasoning. Then indicate for each the degree of confidence (1-low, 5-high) that you have in your conclusion and justification.

1. Bob claims that multiplying any two numbers will always result in an odd number (e.g., 1,3,5,7,9,11,…). Please describe your justification for whether you believe his claim to be true.

Tasks

2. Below is a function that Bob claims is a "prime number generator"—that is, for every numerical input {n=1,2,3,…}, the output is a prime number (i.e., a number not divisible by any number except 1 and itself—examples: 2, 3, 4, 7, 11, 23…). Please describe your justification for whether you believe his claim to be true.

$$p(n) = n^2 - n + 41$$

3. Bob claims that the expression, $\frac{n^2 + n}{2}$, will never result in a decimal for every numerical input {n=1,2,3,…}. Please describe your justification for whether you believe his claim to be true.

Analysis Tool for Coding

	Code	Description	Number
Remove	Flaw	Flawed understanding; mis-interpretation	Remove
External	External	Reasoning linked to external conviction (e.g., just because its true; teacher said so)	0
Inductive/ Empirical Example-based evidence	Naïve	Reasoning linked to small number of cases	1
	Crucial	Reasoning linked to a non-particular case (e.g., deliberate choice is made in test case)	2
	Generic	Reasoning is linked to example as class of cases; generalizations inaccurate or correct but with limited justification	3
	Limitations	Recognizes limitations of examples	3
Deductive	Proof	Logical deductions; correct use of counterexample	4

Examples of Coding Proof

- Flaw (Remove)

2. Below is a function that Bob claims is a "prime number generator" – that is, for every numerical input {n=1, 2, 3, …}, the output is a prime number (i.e., a number not divisible by any number except 1 and itself – examples: 2, 3, 5, 7, 11, 23…). Please describe your justification for whether you believe his claim to be true.

$$p(n) = n^2 - n + 41$$

$p(7) = 7^2 - 4 + 41$
$= 49 - 4 + 41$
$= 45 + 41$
$= 86$

$p(4) = 4^2 - 4 + 41$
$= 16 - 4 + 41$
$= 12 + 41$
✓ $= 53$

Confidence (1-5): 4

Bob is incorrect because if I input the prime number 7, the output is 86 which has more than 2 factors.

Examples of Coding Proof

- External Conviction (Score=0)

2. Below is a function that Bob claims is a “prime number generator” – that is, for every numerical input {n=1, 2, 3, …}, the output is a prime number (i.e., a number not divisible by any number except 1 and itself – examples: 2, 3, 5, 7, 11, 23…). Please describe your justification for whether you believe his claim to be true.

$$p(n) = n^2 - n + 41$$

Yes, Quadratic Formulas always result in prime numbers.

Confidence (1-5): 4

Examples of Coding Proof

- Naïve empiricism (Score=1)

This claim is true because (n=1,2,3) results in 1,3,&6 -no decimal.

$$\frac{n^2+n}{2} \quad \frac{1^2+1}{2} = \frac{1+1}{2} = \frac{2}{2} = 1 \quad \frac{2^2+2}{2} = \frac{4+2}{2} = \frac{6}{2} = 3$$

$$\frac{3^2+3}{2} \quad \frac{9+3}{2} \quad \frac{12}{2} = 6$$

1. Bob claims that multiplying any two odd numbers will always result in an odd number (e.g., 1, 3, 5, 7, 9, 11, ...). Please describe your justification for whether you believe his claim to be true.

$3 \cdot 5 = 15$
$5 \cdot 7 = 35$
$7 \cdot 9 = 63$

Trying different examples I agree with Bob. A product of two ODD number does generate an ODD result.

Confidence (1-5): 5

Examples of Coding Proof

- Crucial Experiment (Score=2)

2. Below is a function that Bob claims is a "prime number generator" – that is, for every numerical input {n=1, 2, 3, …}, the output is a prime number (i.e., a number not divisible by any number except 1 and itself – examples: 2, 3, 5, 7, 11, 23…). Please describe your justification for whether you believe his claim to be true.

$144-12=132$ $\quad p(n)=n^2-n+41$

$p(12)=173$

$p(1)=41$

$p(2)=43$

$p(10)=90+41=131$

Confidence (1-5): 5

I agree with Bob's claim after trying different values I did prove it generates a prime number.

(2) $\dfrac{n^2+n}{2}$ $\quad n=1,2,3$

$=\dfrac{1^2+1}{2} \qquad \dfrac{2^2+2}{2} \qquad \dfrac{3^2+3}{2}$

$=\dfrac{1+1}{2} \qquad =\dfrac{4+2}{2} \qquad \dfrac{9+3}{2}$

$=\dfrac{2}{2} \qquad =\dfrac{6}{2} \qquad \dfrac{12}{2}$

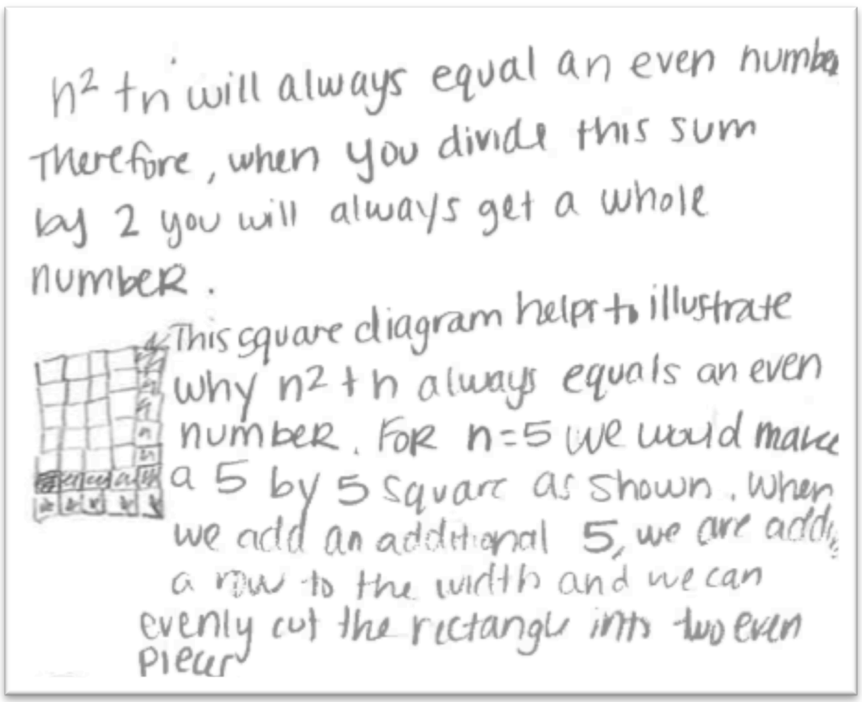
$=1 \qquad =3 \qquad =6 \qquad 6$

Bob's claim seems to be true for the first three numbers

$\dfrac{25^2+25}{2}=325$ It seems to be true. I have tried four different numbers.

Examples of Coding Proof

- Generic Example (Score=3)

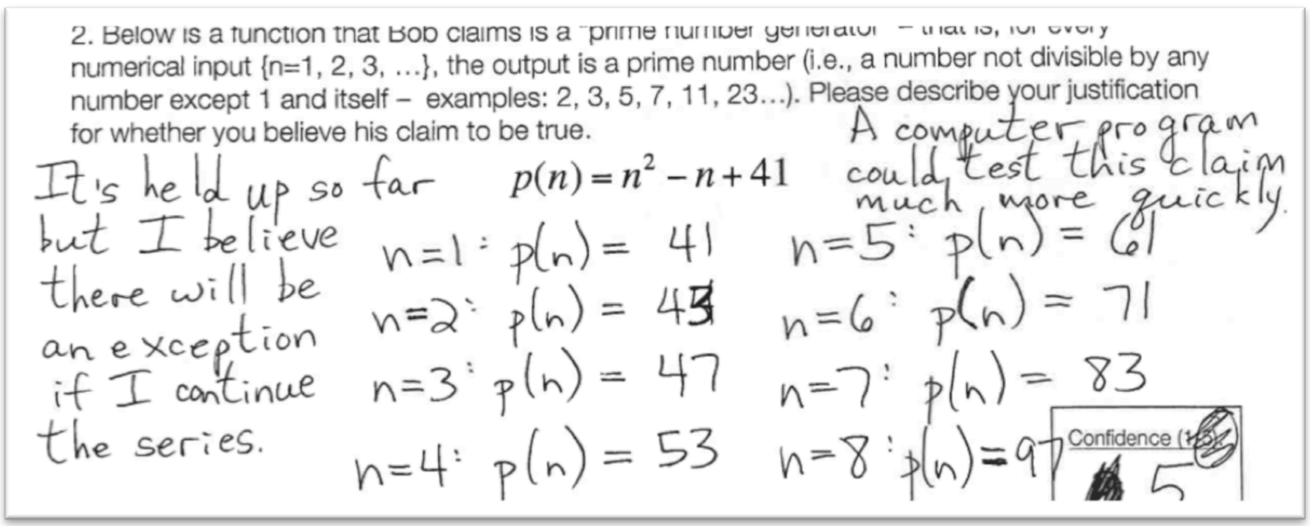


n^2+n will always equal an even number
Therefore, when you divide this sum by 2 you will always get a whole number.

This square diagram helps to illustrate why n^2+n always equals an even number. For $n=5$ we would make a 5 by 5 square as shown. When we add an additional 5, we are adding a row to the width and we can evenly cut the rectangle into two even pieces

Examples of Coding Proof

- Limitations (Score=3)



2. Below is a function that Bob claims is a "prime number generator" – that is, for every numerical input {n=1, 2, 3, …}, the output is a prime number (i.e., a number not divisible by any number except 1 and itself – examples: 2, 3, 5, 7, 11, 23…). Please describe your justification for whether you believe his claim to be true.

$$p(n) = n^2 - n + 41$$

It's held up so far but I believe there will be an exception if I continue the series.

n=1: p(n) = 41
n=2: p(n) = 43
n=3: p(n) = 47
n=4: p(n) = 53
n=5: p(n) = 61
n=6: p(n) = 71
n=7: p(n) = 83
n=8: p(n) = 97

A computer program could test this claim much more quickly.

Confidence (1-5)
5

Examples of Coding Proof

- Thought Experiment (Score=4)

$$\frac{1^2+1}{2} = \frac{2}{2} = 1$$

$$\frac{2^2+2}{2} = 3$$

$$\frac{7^2+7}{2} = \quad \leftarrow 7 \cdot 8$$

↑ even

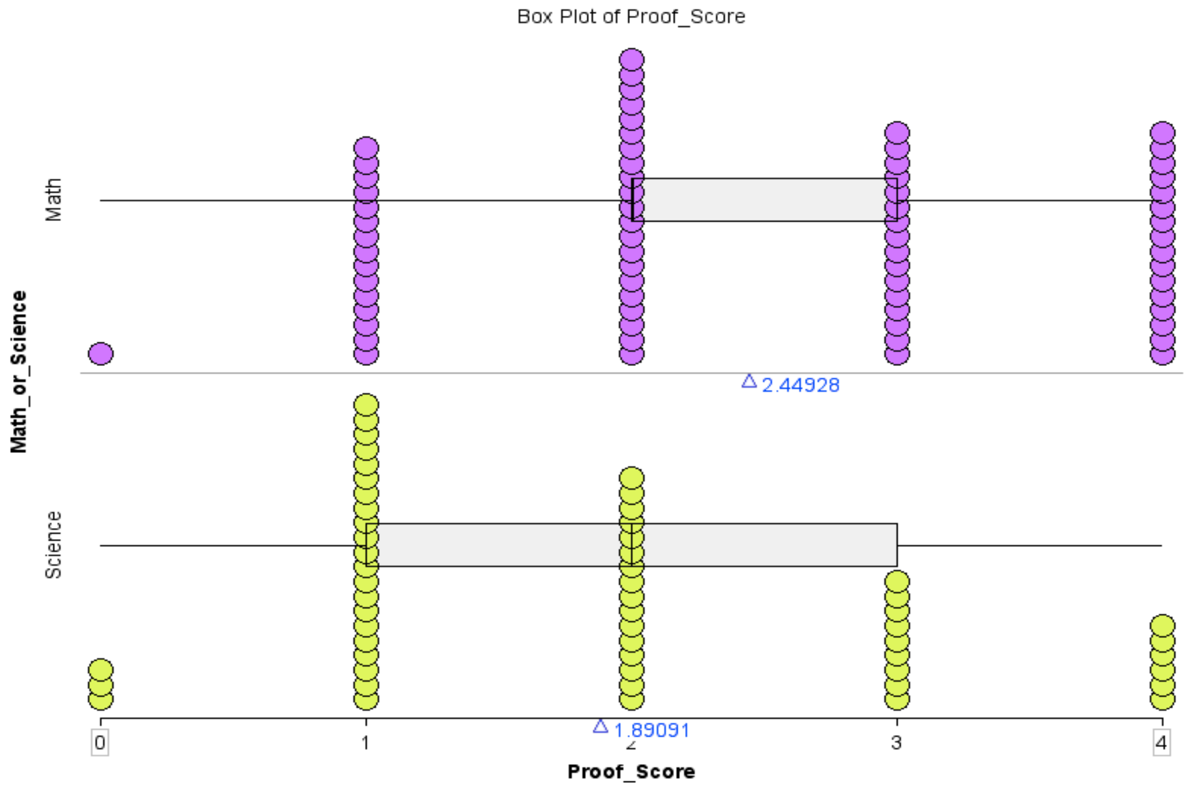
True because an even # squared will be even and adding another even # will still result in an even #. An odd # squared will be odd, but adding one more "n" makes it an even number of odds (resulting in an even #, which is divisible by 2.

Confidence (1-5): 5

Findings

- Over all 3 problems

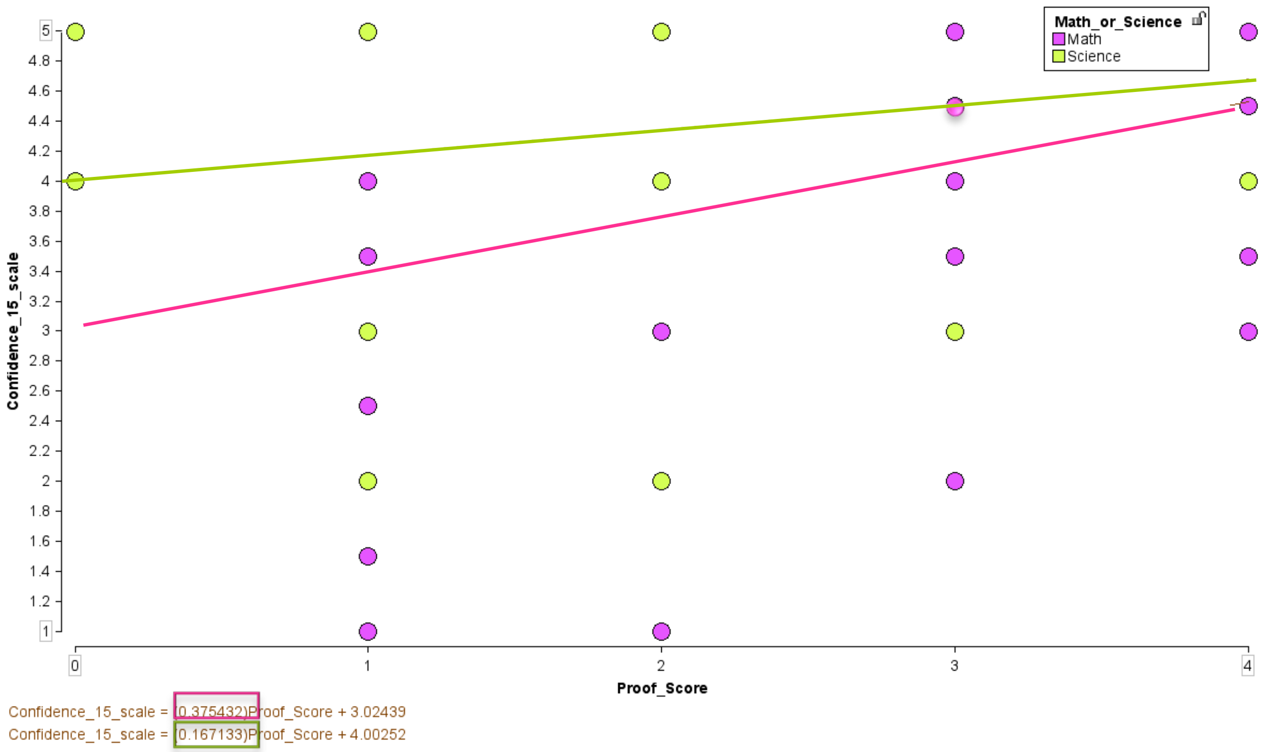
Math teachers, overall, had (statistically significant) higher proof scores



Findings

- Over all 3 problems

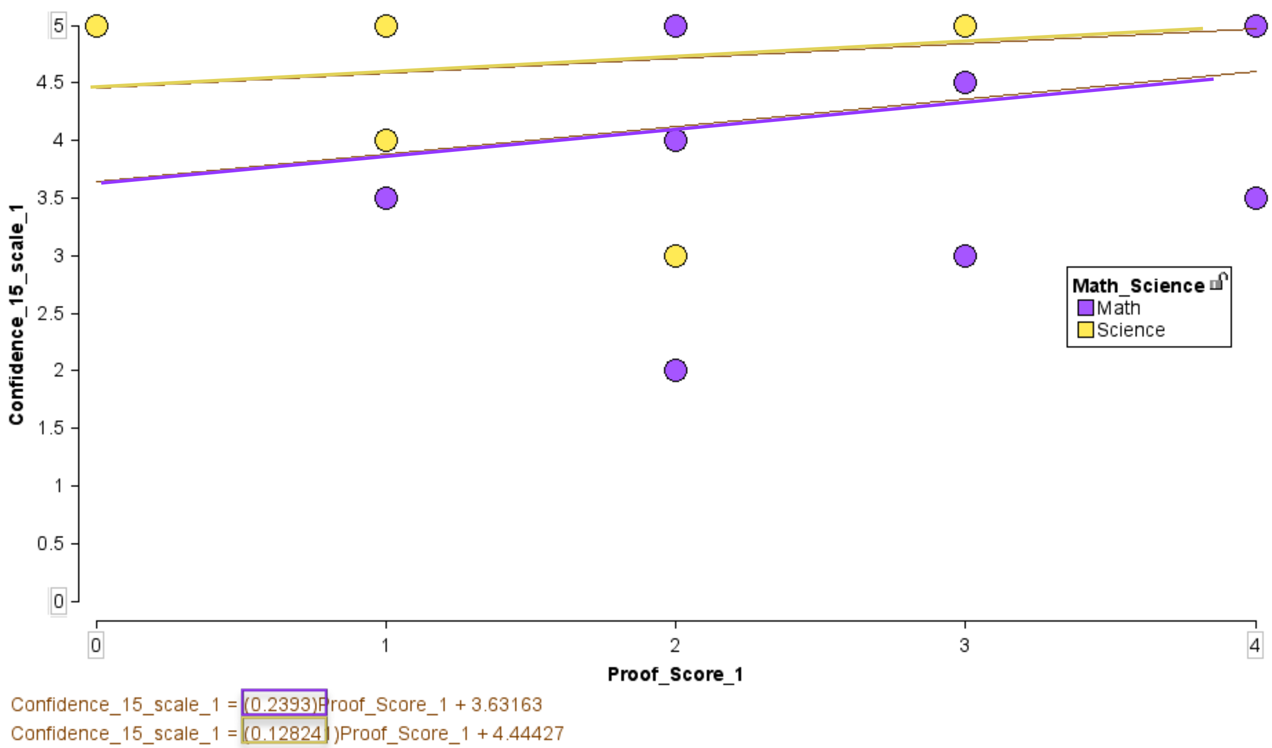
Slope Coefficient:
(probability of having m=0)
Math: p=.006***
Science: p=.171



Findings

Slope Coefficient:
(probability of having m=0)
Math: p=.251
Science: p=.347

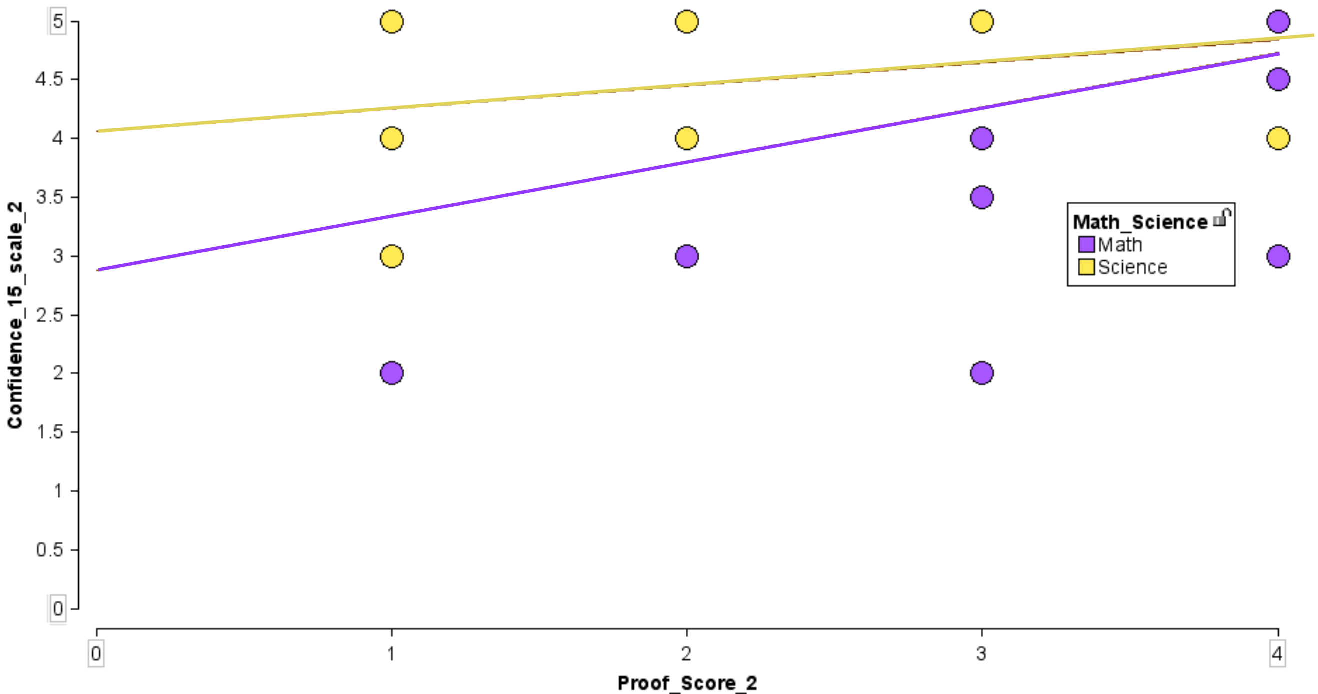
- Problem 1: Product of Odds



Findings

- Problem 2: $(n^2+n)/2$

Slope Coefficient:
(probability of having m=0)
Math: p=.042***
Science: p=.257

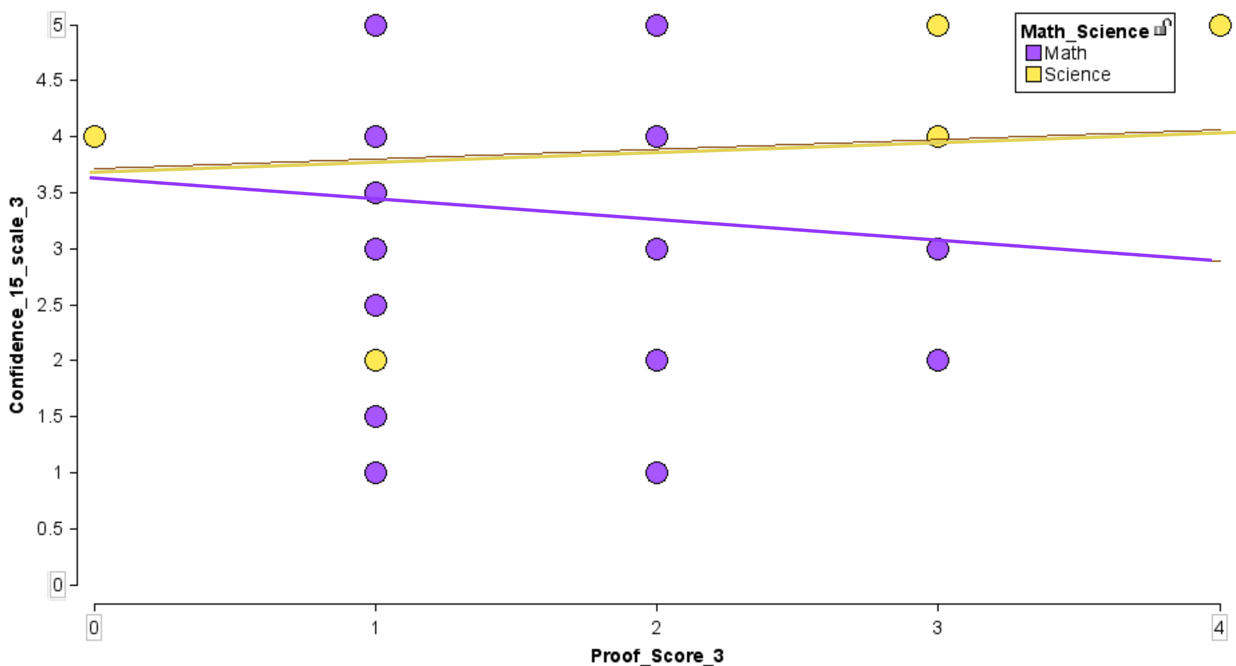


Confidence_15_scale_2 = (0.463959)Proof_Score_2 + 2.86861
Confidence_15_scale_2 = (0.194595)Proof_Score_2 + 4.04517

Findings

Slope Coefficient:
(probability of having m=0)
Math: p=.648
Science: p=.774

- Problem 3: Prime generator



Confidence_15_scale_3 = (-0.185364)Proof_Score_3 + 3.61786
Confidence_15_scale_3 = (0.086805)Proof_Score_3 + 3.70264

Quotes

"I think scientists and mathematicians add new knowledge in essentially the same manner."

"I don't think there are any major differences."

"The differences are not major."

"I do no think that there are major differences between how scientists and mathematicians add new knowledge to their fields."

Quotes about Math

“Mathematicians tend to validate all of their findings using mathematical models, thereby offering mathematical "proofs". In science this is also done, but observation plays a larger role.”

“Mathematicians ideas do not have to correspond to any physical reality and thus are not subject to experimental verification.”

Quotes about Science

"Scientists test their ideas through observation and experimentation."

"Scientists add knowledge by observing natural phenomena, asking questions about those phenomena, then collect data and look for some pattern in the data."

Conclusions

- Significant difference between math and science teachers' reasoning on mathematics tasks
- Significant difference between math and science teachers' confidence in inductive reasoning as sufficient evidence
- Little evidence that teachers' distinguish between the different modes of reasoning in mathematics and science

Implications

- There is disciplinary knowledge, specific to each discipline (Math, Science, Technology, Engineering) that cannot and should not be lost if we move toward more integrated STEM education.
- Need to make sure that teachers who engage in an integrated STEM curriculum are aware of the different modes of reasoning and validation in each discipline (particular Math and Science)
- If STEM Integration is a goal, we need to make sure that each discipline is still treated with integrity

Thank you!

Questions? Comments?

Nick Wasserman, nwasserman@smu.edu
Dara Williams-Rossi, drossi@smu.edu